Training cascaded networks for speeded decisions using a temporal-difference loss

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Abstract

Although deep feedforward neural networks share some characteristics with the primate visual system, a key distinction is their dynamics. Deep nets typically operate in sequential stages wherein each layer fully completes its computation before processing begins in subsequent layers. In contrast, biological systems have cascaded dynamics: information propagates from neurons at all layers in parallel but transmission is gradual over time. In our work, we construct a cascaded ResNet by introducing a propagation delay into each residual block and updating all layers in parallel in a stateful manner. Because information transmitted through skip connections avoids delays, the functional depth of the architecture increases over time and yields a trade off between processing speed and accuracy. We introduce a temporal-difference (TD) training loss that achieves a strictly superior speed-accuracy profile over standard losses. The \textsc{CascadedTD} model has intriguing properties, including: typical instances are classified more rapidly than atypical instances; \textsc{CascadedTD} is more robust to both persistent and transient noise than is a conventional ResNet; and the time-varying output trace of \textsc{CascadedTD} provides a signal that can be used by ‘meta-cognitive’ models for OOD detection and to determine when to terminate processing.

1. Introduction

A synergistic relationship has long existed between theories of human vision and deep neural networks. Deep nets have been used as a model of human vision (Kriegeskorte, 2015; Lindsay, 2020). And deep nets have been fruitfully informed by neuroscience. The most compelling such example is of course convolutional networks, which have adopted properties of primate cortical neuroanatomy including a hierarchical layered architecture, local receptive fields, and spatial equivariance (Fukushima, 1980). Despite the synergy, key properties of biological information processing systems have been set aside in the design of neural networks. In this article, we consider four fundamental but neglected properties. First, the brain consists of dedicated hardware and all neurons in all layers update continually and in parallel. Second, information transmission between neurons is relatively slow. Third, unrefined and possibly incomplete neural state in one layer is transmitted even as the state evolves and is refined. Fourth, although cortical vision is hierarchically organized, many skip-layer connections exist (Figure 1), allowing for multiple paths through which information flows. Taken together, these properties have a fundamental consequence for network dynamics: the architecture produces a speed-accuracy trade off in which the initial output occurs rapidly but can be inaccurate, and the output improves gradually over time. Following McClelland (1979), we refer to such an architecture as \textit{cascaded}. Cascaded dynamics contrast sharply with the dynamics of standard feedforward nets, which operate in sequential stages and each layer completes its computation before subsequent layers begin processing.

In this article, we construct cascaded networks by introducing propagation time delays in deep feedforward nets and treat the net as massively parallel such that the states of all units across all layers are updated simultaneously and iteratively. We focus on the ResNet architecture (He et al., 2016) and we introduce a propagation delay into each

![Figure 1. Simplified scheme for cortical organization of primate visual system demonstrating hierarchical and parallel systems with skip connections bypassing cortical regions. Reprinted from Cox & Maier (2015).](image-url)
2. Related Work

**Cortical parallelism.** Primate visual cortex is organized in a hierarchical fashion but its connectivity is not strictly sequential layer-to-layer. Felleman & Van Essen (1991) identified 305 connections amongst 32 visual and visual-association areas in the primate cerebral cortex, showing a degree of connectivity between visual regions close to 40%. This massive connectivity coupled with neurophysiological principles (Gerstner et al., 1997) imbues the system with massive parallelism, which neural networks lack due to the conventional usage of discrete stage processing.

**Prior research on cascaded models.** McClelland (1979) characterizes human mental computation from a psychological perspective in terms of a staged architecture with the stages operating in cascade. The stages accumulate information with leaky integrators. In this view of information processing, the stages operate simultaneously, passing along partial information as it becomes available rather than completing one stage before the next stage begins operation. In recent years, the term *cascade prediction* has been used to describe the prediction of information cascades in social networks (Banerjee, 1992; Huang & Chen, 2015); we mention this unrelated work only to distinguish it from our topic. The ‘cascaded’ moniker has also been applied to work on *any-time prediction*, which assumes sequential operation of layers such that after $t$ steps, $t$ layers have been activated, but at each step a prediction is made from the last activated layer (Hu et al., 2019), as well as to training schemes that learn one layer of the network at a time, yielding a model trained in cascade (Marquez et al., 2018). In our terminology, this framework would be considered as *sequential*. We are aware of only two deep-learning investigations of cascaded models of the form we describe. Both are focused on sequential processing, where the model state from the previous input (e.g., video frame) can be useful for effi-
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3. Cascaded Deep Networks

Many modern deep architectures—including ResNet (He et al., 2016), Highway Nets (Srivastava et al., 2015), DenseNet (Huang et al., 2017), U-Net (Ronneberger et al., 2015)—incorporate skip connections that bypass strictly layered feedforward connectivity, analogous to the architecture of visual cortex (Felleman & Van Essen, 1991). Under the biological assumption that signals transmitted through a neural layer are delayed relative to signals that bypass the layer, we construct a cascaded model using ResNets by introducing a novel computational component that delays the transmission of signals from the output of each computational layer, denoted Δ in Figure 2. Because these delays extend processing in time, the hidden states require time index. The input to ResNet block \( i \) at time \( t \) is denoted \( z_{t,i} \). The block transforms this input via the residual transform, yielding \( z'_{t,i} = F(z_{t,i}) \). We conceive of \( \Delta \) as a tapped delay-line memory of the transform history, \( Z'_{t,i} = [z'_{t,i}, z'_{t-1,i}, \ldots, z'_{1,i}] \), which is convolved with a temporal kernel \( \kappa \) to produce the block output

\[
z_{t,i+1} = \text{ReLU} \left( z_{t,i} + Z'_{t,i} \kappa \right).
\]  

With kernel \( \kappa = [1, 0, 0, \ldots, 0] \), we recover the standard ResNet in which communication between layers is instantaneous. We consider two kernels to introduce time delays. With \( \kappa = [0, 1, 0, 0, \ldots, 0] \), a discrete one-step delay is introduced (OSD for short). With \( \kappa = (1 - \alpha) [1, \alpha, \alpha^2, \alpha^3, \ldots] \), we obtain exponentially weighted smoothing (EWS for short), with larger \( \alpha \in [0, 1) \) producing slower transmission times. Note that both of these special kernels have efficient implementations. The OSD kernel can be implemented with a one-element queue. The EWS kernel can be implemented by an incremental update and a finite (one-step) state vector:

\[
Z'_{t,i} \kappa = \alpha Z'_{t-1,i} \kappa + (1 - \alpha) z'_{t,i}.
\]

We use the OSD kernel for training all models. All experiments use a ResNet-18, which has 9 residual blocks and hence 9 time delays. We also add a time delay to the output of the model’s first convolutional layer. Consequently, with the OSD kernel, the cascaded model requires 10 updates for the output to reach asymptote. At asymptote, the cascaded model is guaranteed to produce the same output as would the standard \textit{Sequential} model with the same weights.

To obtain a finer granularity of time at evaluation, some simulations switch to the EWS kernel with \( \alpha = 0.9 \). The model’s temporal dynamics are qualitatively similar whether OSD or EWS is used, but using EWS allows us to better distinguish individual examples in terms of their temporal dynamics. EWS with \( \alpha = 0.9 \) requires about 70 steps for the output to asymptote. The choice of \( \alpha \) over a had no impact on our findings, as long as it slowed transmission.

3.1. Training Cascaded Networks with TD(\( \lambda \))

Our \textit{Sequential} model is a standard ResNet, which we train as detailed in Appendix A. Our \textit{CascadedSeq} model uses the weights from the standard ResNet but at evaluation is run as a cascaded model. Our \textit{CascadedTD} model is trained from scratch using the OSD kernel with the goal of producing the correct output as quickly as possible. \textit{CascadedTD} is unrolled for \( T \) steps and trained with back propagation through time, where \( T \) is the number of delay components in the model. To encourage correct outputs sooner, we use the methods of temporal differences (TD; Sutton, 1988). Readers may associate TD methods with reinforcement learning because TD methods have traditionally been used to predict future rewards. However, TD methods are fundamentally designed for supervised learning. We use TD to predict a future outcome—the correct classification of an image—from a sequence of successively more informative states—the cascaded information flowing through the ResNet.

TD(\( \lambda \) ) specifies a target output \( y_t \) at each time \( t \in \{1, \ldots, T\} \) based on the model’s actual output \( \hat{y}_{t+i} \) at future times \( t+i \) for \( i > 0 \), and the eventual outcome or true target, \( y_{true} \):

\[
y_t = (1 - \lambda) \sum_{i=1}^{T-t} \lambda^{i-1} \hat{y}_{t+i} + \lambda^{T-t} y_{true},
\]

where \( \lambda \in [0, 1] \) is a free parameter that essentially specifies the time horizon for prediction. TD(1) predicts the eventual outcome at each step; TD(0) predicts the model’s
output at the next step (and the eventual outcome at the final step). Given target \( y_t \) and actual output \( \hat{y}_t \), we specify a cross-entropy loss, \( \mathcal{L} = \sum_{t=0}^{T} H(y_t, \hat{y}_t) \), where \( H(p, q) \) is the cross-entropy. Note that \( y_t \) must be treated as a constant, not as a differentiable variable; TensorFlow and Jax require a `stop_gradient`, PyTorch instead uses `requires_grad=False`. Although Equation 2 requires knowledge of all subsequent network states, the beauty of TD methods is that this loss can be computed incrementally (Appendix A.2). The edge cases \( \text{TD}(0) \) and \( \text{TD}(1) \) have particularly trivial implementations. Past research has always used \( \text{TD}(1) \) for specifying intermediate targets, but we will show that \( \text{TD}(1) \) is suboptimal because the model is penalized for being unable to classify correctly at the earliest steps.

4. Results

4.1. TD(λ) Training

We conducted a sweep over hyperparameter \( \lambda \) to determine its effect on asymptotic accuracy of \( \text{CascadedTD} \). Figure 3a shows results from five replications of \( \text{CascadedTD} \) on CIFAR-100. The hyperparameter has a systematic effect, consistent with classic studies with linear models (Sutton & Barto, 2018, Chapter 12). We also conducted simulations with CIFAR-10 and TinyImageNet that produce clear inverted-U curves for accuracy versus \( \lambda \) (Figures A.3a, A.4a). Importantly, the choice of \( \lambda = 1 \), which is the standard approach to training models to obtain an accurate response sooner, obtains the poorest performance for all three data sets, significantly worse than \( \lambda < .5 \). The essential explanation is that larger \( \lambda \) penalize the network for behavior it does not have the capability to achieve: obtaining the asymptotic prediction at the earliest time steps. To paraphrase the classic illustration of TD from Sutton (1988), if the task is predicting the weather on December 31, no model can predict as accurately on December 1 as on December 30. Selecting \( \lambda < 1 \) shortens the prediction horizon; \( \lambda = 0 \) corresponds with requiring a prediction only of the weather on the next day.

For CIFAR-100, \( \text{CascadedTD} \)’s asymptotic accuracy for \( \lambda = 0 \) is 67.5% (SEM 0.1%), which is significantly better than the accuracy of 65.6% (SEM 0.1%) achieved by both \( \text{Sequential} \) and \( \text{CascadedSeq} \), whose weights are based on the standard cross-entropy loss. This boost in accuracy with TD training is quite surprising considering that TD training imposes constraints on the output at each time, not just the asymptotic output. The accuracy boost suggests that TD training encourages the network to reorganize its knowledge in a manner that is beneficial for generalization, a point that we explore shortly. However, no boost is observed for CIFAR-10 (TD: 91.7%, SEM 0.2%; standard: 91.9%, SEM 0.1%) and a slight drop is observed for TinyImageNet (TD: 52.3%, SEM 0.07%; standard: 52.5%, SEM 0.06%), so it remains to be determined whether TD training might be a boon not only for cascaded models but also for models that will be run in the sequential layer-by-layer mode.

Because we focus on CIFAR-100 in the main article, and because TD(0) has a particularly trivial implementation relative to \( \lambda > 0 \) (there is no need for eligibility traces), we use TD(0) in all following simulations.

4.2. Speed-Accuracy Trade Offs

Under the dedicated-hardware assumption that all layers in a model can update in parallel at each time step, we can compute accuracy of response as a function of time. Figure 3b presents accuracy as a function of time on CIFAR-100. \( \text{Sequential} \) yields no meaningful output until the final time step, as it does not exploit hardware parallelism. \( \text{CascadedSeq} \), obtained by cascading the \( \text{Sequential} \) model at test-time, does produce a speed-accuracy trade off but it takes about four updates to begin producing meaningful output. In contrast, \( \text{CascadedTD} \), trained as a cascaded model using TD(0) to encourage the network to quickly yield correct responses, shows a strictly superior speed-accuracy profile over \( \text{CascadedSeq} \). Similar results are obtained for CIFAR-10 and TinyImageNet (Figures A.3b, A.4b).
Beyond investigating the time course of fine-grain classification, we also examined coarse-grain classification. Forming twenty superclasses from the 100 fine-grain classes of CIFAR-100, as specified in Krizhevsky et al. (2009), we examined the probability of correct coarse-grain classification conditional on incorrect fine-grain classification. Zamir et al. (2017) refer to this probability as taxonomic compliance, which reflects information being transmitted about coarse category even when the specific class cannot be determined. As shown in Figure 3c, taxonomic compliance rises faster for CASCADED TD than for CASCADED SEQ. Whereas chance compliance is .05, CASCADED TD achieves a compliance probability of .35 after 2 steps. CASCADED SEQ requires 8 steps to achieve the same performance.

To understand the implication of Figures 3b,c for how the ResNet organizes knowledge, we need to discuss the flow of information in the cascaded model. At time 1, only the first residual block has received meaningful input, and the output is therefore based only on this block’s computation. At time 2, all higher residual blocks have received input from block 1, and the output is therefore based on all blocks’ computations, though blocks 2 and above have deficient input. At each subsequent time, all blocks are receiving meaningful input, but it is not until time $t$ that block $t$ has reached its asymptotic output because its input does not stabilize until $t - 1$ (Figure A.1 depicts this flow of information over time). Because it takes $t$ steps for information to completely propagate through $t$ layers, one can argue that the network’s functional depth increases over time. That CASCADED TD makes classification decisions above chance after 1 step indicates that the functional depth of the model has flattened relative to CASCADED SEQ. In essence, TD training encourages the model to behave more like a WideResNet (Zagoruyko & Komodakis, 2016) than a standard deep ResNet.

### 4.3. Stratifying Instances by Selection Latency

Having examined the response profile of our models over an evaluation set, we now turn to analyzing the response to individual instances. Specifically, we ask about the time course of reaching a classification decision. We define the selection latency for an instance to be the minimum number of steps required for the classifier to reach a confidence threshold for a particular class and maintain that level going forward, i.e., $\min \{ t \mid \exists j \mid y_{t', j} > \theta \forall t' \geq t \}$, where $y$ is the model output, $j$ is an index over classes, and $\theta$ is the threshold. The selection latency does not specify whether or not the chosen class is correct. We picked a threshold such that only $\sim 10\%$ of the test examples failed to reach threshold, $\theta = 0.83$; reported results are robust to this $\theta$.

Having determined a selection latency for most evaluation instances, we now consider key factors that determine the latency. Our basic finding is that selection latency is inversely related to instance prototypicality: the more prototypical instances are classified quicker.

Figure 4 shows sets of instances of various classes determined by the selection latency of the CASCADED TD and CASCADED SEQ models (left and right halves of the figures, respectively). In each row are instances from a particular class. The leftmost three images have the lowest selection latency (easy to classify), the rightmost three images have the highest selection latency (hard). We note the homogeneity of the low-latency images for CASCADED TD, where most objects are against a solid background with no clutter in the image; in contrast, the high-latency images have complex backgrounds, such as the fish of varying shapes and colors. The low-latency images might be described as prototypical and the high-latency images as outliers. Turning to CASCADED SEQ, no such pattern is evident; the instances do not appear to stratify by complexity or typicality. In the rest of this section, we formalize the notion of prototypicality and explore its relationship to selection latency. We define three measures of prototypicality: for each of these measures, a larger value means more prototypical.

- **Centrality.** We compute the cosine distance of an instance’s embedding (the penultimate layer activation) and the target-class weight vector. The larger this quantity, the better aligned the two vectors are. Because the weight vector will tend to point near the center of class instances, the cosine distance is a measure of instance centrality.

- **C-score.** Jiang et al. (2020) describe an instance-based measure of statistical regularity called the C-score. The C-score is an empirical estimate of the probability that a network will generalize correctly to an instance if it is held out from the training set. It reflects statistical regularity in that an instance similar to many other instances in the training set should have a high C-score.

- **Human labeling consistency.** Peterson et al. (2019) collected human labels on images. Most images are consistently labeled, but some are ambiguous. The negative entropy of the response distribution indicates inter-human labeling agreement. Presumably consistently labeled in-
Table 1. Spearman rank correlations between three measures of instance prototypicality and selection latency. Larger coefficient indicates faster responses for more prototypical instances.

<table>
<thead>
<tr>
<th>Measure</th>
<th>Spearman’s $\rho$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>CASCADEDSEQ</td>
</tr>
<tr>
<td>centrality</td>
<td>0.140</td>
</tr>
<tr>
<td>C-score</td>
<td>0.326</td>
</tr>
<tr>
<td>human consistency</td>
<td>0.153</td>
</tr>
</tbody>
</table>

All three measures are available only for the CIFAR-10 training set. Consequently, we ran 10-fold cross validation on the training set, assessing the correlation based on the held out images in each fold. To obtain some granularity on the selection latency, we use the EWS kernel.

Table 1 presents the correlation—Spearman’s $\rho$—between the three prototypicality measures and negative selection latency for CASCADEDSEQ and CASCADEDTD. A positive coefficient indicates shorter latency for prototypical instances. The coefficient is reliably positive ($p < .001$) for each of the three typicality measures and both models. However, CASCADEDTD obtains reliably higher correlations on two of the three measures than CASCADEDSEQ ($p < .001$). They are not significantly different on the consistency measure ($p = .29$). Thus, by these quantitative scores, the TD training procedure leads to better stratification of instances by typicality, in line with the qualitative results presented in Figure A.2. Why does TD training distinguish instances based on prototypicality? Intuitively, a prototypical instance shares features with many other class instances. Because these features are frequent in the data set, the TD loss focuses on classifying instances with those features rapidly.

4.4. Effects of Time-Varying Inputs

In this section, we explore the effects of time-varying input (TVI) in the form of noise perturbations of a single image. Because the external environment can change more rapidly than any snapshot of the environment can be processed, the cascaded model will necessarily integrate signals from multiple snapshots. We observe a consistent benefit of CASCADEDTD over CASCADEDSEQ or SEQUENTIAL.

Figure 5 shows the six types of noise we consider on CIFAR-10 images. From left-to-right, top-to-bottom: (1) Focus: a 16 × 16 foveated patch randomly placed within the image, where regions outside of the patch are Gaussian blurred; (2) Perlin: gradient noise randomly applied to 40% of the image; (3) Translation: random shifts ±8 pixels in $(x, y)$ on a reflection-padded image; (4) Occlusion: a 16 × 16 occluding patch randomly placed within the image; (5) Resolution: random downsampling by factors of $0 \times 2$, $2 \times$, or $4 \times$ via average pooling followed by $k$-nearest upsampling to recover the original dimensionality of $32 \times 32$. (6) Rotation: random rotations ±60° on a reflection padded image. Four of the perturbations are lossy; only rotation and translation are roughly information preserving.

For each noise type, a SEQUENTIAL and a CASCADEDTD model are trained with the corresponding image transformation type as a data augmentation. (CASCADEDSEQ inherits weights from SEQUENTIAL.) The training details of Appendix A are followed, except that we discard the 8 × 8 block data augmentation in order to avoid biasing the models toward the Occlusion noise transformation. We evaluate the cascaded models with the OSD kernel to allow for a comparison of cascaded models with SEQUENTIAL.

In a first experiment, TVI.1, the six noise types are persistent: the same noise source transforms the image on each time step over the entire time course of processing. We assess asymptotic (final step) accuracy on evaluation-set examples. In this setting, we treat the SEQUENTIAL model as being capable of computing a complete input-to-output activation pass at every time step, as opposed to assuming that we are operating on truly parallel hardware, which provides the cascaded models a 10-fold advantage.

Given the stateful nature of the CASCADEDTD model, we hypothesize that it will outperform SEQUENTIAL and CASCADEDSEQ for lossy noise types due to being capable of integrating information over time. The results in Table 2 support our hypothesis; the asymptotic test-set accuracy of the CASCADEDTD model is significantly higher than that of the SEQUENTIAL and CASCADEDSEQ models for the four lossy noise transformations, suggesting that training with TD encourages the cascaded model to integrate information over time allowing for more robust representations lead to improved accuracy under lossy, time-varying inputs.

Table 2. TVI.1 experiment on persistent noise. Highlight indicates best asymptotic performance for a given noise type.

<table>
<thead>
<tr>
<th>TVI.1</th>
<th>Asymptotic Model Performance</th>
</tr>
</thead>
<tbody>
<tr>
<td>Noise</td>
<td>SEQUENTIAL</td>
</tr>
<tr>
<td>Focus</td>
<td>84.27 ± 0.06</td>
</tr>
<tr>
<td>Occlusion</td>
<td>86.26 ± 0.08</td>
</tr>
<tr>
<td>Perlin</td>
<td>85.18 ± 0.03</td>
</tr>
<tr>
<td>Resolution</td>
<td>84.53 ± 0.07</td>
</tr>
<tr>
<td>Rotation</td>
<td>89.11 ± 0.04</td>
</tr>
<tr>
<td>Translation</td>
<td>87.55 ± 0.12</td>
</tr>
</tbody>
</table>
Conversely, **Sequential** is best on information preserving transformations such as **Translation** and **Rotation** because it is performing a full inference pass on the input whereas the cascaded models are performing a single update step.

In a second experiment, **TVI.2**, we present the noise-free input for 10 time steps (sufficient for the cascaded models to reach asymptote), apply one of the six noise transforms for $N$ time steps, and then present the noise-free input for another 10 steps, allowing the model to return to its asymptotic state (Figure 6). We run five trials per condition for each $N \in \{1, 2, 3, 4, 5, 6\}$. Performance is evaluated as the drop-in-integrated-performance, $\text{DIP} = \hat{y}_T - \frac{1}{B} \sum_{t=1}^{T} \hat{y}_t$, where $T$ is the total time steps in the simulation, $B$ is the on-set time of the noise transformations, and $\hat{y}_t$ is the model’s target-class confidence at time step $t$. **DIP** indicates how quickly a model can recover from noise perturbations.

On the whole, **CascadedTD** is more robust to transient perturbations than **Sequential** and **CascadedSeq** (Table 3). **Sequential** has a disadvantage in being unable to perform noise cancellation by combining signals across frames, yet it has a massive advantage in performing ten sequential updates per time step, versus one parallel update for the cascaded models. While both **CascadedTD** and **CascadedSeq** smooth responses over frames, **CascadedTD** performs better, indicating that beyond smoothing, TD training orchestrates the integration of image-specific perceptual information. This integration matters more for lossy transformations, where information integration is essential.

### 4.5. Meta-cognitive Inference

In this section, we consider the hypothesis that temporally intermediate outputs from cascaded networks can provide additional signals to improve performance. We term this **metacognition**, by reference to human abilities to reason about our reasoning processes. For example, we might want to greet an acquaintance by name, but if their name comes to mind only after a long lag, we might lack the confidence to speak their name out loud and instead choose a less personal greeting. To map this scenario to our present setting, we explore the idea of using the temporal trace of output from the cascaded model to train a separate classifier, which we call **MetaCog**, to make judgments about the performance of the cascaded model. The particular judgments are OOD detection and deciding when to commit to a response.

#### 4.5.1. Out-of-Distribution Detection

In this set of experiments, we utilize the output of **CascadedTD** to train **MetaCog** discriminatively for OOD detection and show that it significantly outperforms models trained using only the asymptotic **CascadedTD** output. **MetaCog** is a multi-layer perceptron with a 256-unit hidden layer and a sigmoidal output unit for binary prediction: 1 or 0 for in- or out-of-distribution instances, respectively. CIFAR-10’s validation set serves as the in-distribution training examples, whereas the validation sets of TinyImageNet, LSUN, and SVHN serve as OOD training examples, with crop and resize variations to TinyImageNet and LSUN following Liang et al. (2017); see details in Appendix B.1. One **MetaCog** model is trained per (in-, out-of-distribution) dataset pairing—e.g., (CIFAR-10, SVHN)—and input representation type (discussed below). The respective test set is used for evaluation. The **CascadedTD** output serves as input to **MetaCog** and is represented in one of four ways: (1) the confidence of the most probable class, known as the **max softmax prediction** or MSP, (2) entropy of the class posterior distribution, (3) the class posterior distribution, or (4) the logit representation of the posterior. We investigate whether feeding the output of all time steps to **MetaCog** leads to improved prediction relative to feeding only the final asymptotic output. In a standard feedforward net, only the latter information is available.

A baseline metric is computed directly from the max softmax predictions of the **CascadedTD** model, whereas the meta-cognitive metrics are computed by averaging accuracy over dataset pairings. Following Liang et al. (2017), we assess OOD performance with **AUROC**, the Area Under the Receiver Operating Characteristic curve, and **FPR @ 95% TPR**, the false positive rate at 95% true positive rate.
Using the temporal dynamics significantly improves OOD detection performance on both metrics regardless of representation used (Figure 7). Additionally, the logit representation obtains the best performance of all the representations considered (see Table B.3 for quantitative results).

4.5.2. Response Initiation

Lastly, we explore the use of MetaCog for deciding when to initiate a response. That is, given the output sequence from CascadedTD up to time \( t \), should the most probable class label at \( t \) be chosen, or should processing continue at least through \( t + 1 \)? We train a MetaCog model and show significant improvements in the model’s speed-accuracy trade off. Because we need to provide a varying-length input history to MetaCog, we use a GRU recurrent net (Cho et al., 2014). The GRU ingests the sequence of logits and produces a scalar output. MetaCog is trained with a cross-entropy loss summed over time steps to output 1 if CascadedTD’s current class-label prediction matches its asymptotic prediction, otherwise 0. See Appendix B.3 for additional training details.

For evaluation, we test when MetaCog reaches a given threshold, \( \theta \), and determine model mean accuracy and stopping time for that \( \theta \). Varying \( \theta \), we obtain a speed-accuracy curve which can be compared to CascadedTD’s original curve (Figure 8). The outcome is a significant decrease in latency for a given level of accuracy, e.g., the latency is approximately cut in half for an accuracy of 85%. The same analysis was conducted with CascadedSeq, and we found a similar but smaller speed up (Figure A.5). Nonetheless, CascadedTD remained more efficient than CascadedSeq.

5. Discussion

Our exploration of the temporal dynamics of cascaded deep networks has addressed a range of topics, including: speed-accuracy trade offs, understanding what makes images ‘easy’ (classified quickly) and ‘hard’ (classified slowly), moderating time-varying input noise, and demonstrating that the temporal trace of a network’s output provides an additional signal that can be exploited to improve information processing. We hope to have convinced readers that cascaded models have interesting properties and are a promising avenue of exploration. We see three directions in which cascaded nets have particular potential.

• For neuroscientists using deep nets as a model of human vision, cascaded nets are a better approximation to the dynamics of the neural hardware. The properties we investigate—neurons operate in parallel, neurons are stateful, and neurons are slow to transmit information—seem likely to have a critical impact on the nature of cortical computing. As one simple illustration, cortical feedback processes are often posited to be critical for explaining differences in processing efficiency of visual stimuli (e.g., Kar et al., 2019; Spoerer et al., 2017), but we have shown that these difference might well be explained by feedforward cascaded dynamics.

• For hardware researchers, cascaded nets are a possible direction for the future design of AI hardware. It is a direction quite unlike modern GPUs and TPUs, one that exploits massively parallel albeit slow and possibly noisy information processing. Our success in showing strong performance from cascaded models, as well as a training procedure to obtain quick and accurate responses, should encourage researchers in this hardware direction.

• For AI researchers, even those who care nothing about cascaded models per se, cascaded models offer two innovations. First, they offer a promising and unusual way to train feedforward models. One can take a feedforward model, turn it into a cascaded model for training with TD methods, and then run it in Sequential mode. Sequential mode will obtain the same outcome as the asymptotic output of a cascaded model. We’ve shown that TD training can improve asymptotic model accuracy due to inductive biases it imposes on the organization of representations. Further investigations will be needed to understand the conditions under which this improvement...
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occurs. Second, our explorations of metacognition are, to the best of our knowledge, a new direction for the field, perhaps because the temporal dimension is critical to obtaining a representation that is rich enough that it can be mined for insight into the operation of a network.

References


A. Experiment Details

A.1. **Sequential and Cascaded TD Experiment Details**

For all **Sequential** and **Cascaded TD** models, we used a ResNet-18 for CIFAR-10, CIFAR-100, and TinyImageNet datasets. The models were trained using data parallelism over 8 GPUs (see §A.4 for infrastructure details), with the model on each GPU using a batch size of 128. SGD with Nesterov momentum, an initial learning rate of 0.1, weight decay of 0.005, and momentum of 0.9 was used to optimize a softmax cross-entropy loss for **Sequential** and a temporal difference cross-entropy loss for **Cascaded TD**. All models were trained for 200 epochs. The training datasets were split (class-balanced) as 90-10 train-validation, where the validation splits were held out for downstream tasks, such as training **MetaCog** models (see §4.5). For **Cascaded TD**, the batch normalization layer must be augmented such that running means and variances are tracked independently for each timestep. At run-time, if the maximum number of timesteps used during training is exceeded, as occurs with the EWS kernel, the final timestep statistics of the batch normalization layers are used for all subsequent timesteps.

A.2. Incremental TD Formulation and TD(λ) Sweep

The incremental formulation of TD as follows:

\[ y_t^\lambda = (1 - \lambda) \sum_{n=1}^{\infty} \lambda^{n-1} y_{t+n} \]

with

\[ y_{t+n} = \begin{cases} \hat{y}_{t+n} & \text{if } t + n \leq T \\ y & \text{otherwise} \end{cases} \]

\[ = (1 - \lambda) \sum_{n=1}^{T-t} \lambda^{n-1} \hat{y}_{t+n} + (1 - \lambda) \sum_{n=T-t+1}^{\infty} \lambda^{n-1} y \]

\[ = (1 - \lambda) \sum_{n=1}^{T-t} \lambda^{n-1} \hat{y}_{t+n} + \lambda^{T-t} y \]

Here, \( T \) is the number of steps to asymptote, \( y \) is the target output, and \( \hat{y}_t \) is the actual output at step \( t \in \{1, \ldots, T\} \).

Figure A.2. CIFAR-10 examples of low and high selection latency resulting from **Cascaded TD** (left) and **Cascaded Seq** (right).

Note, if \( \lambda = 0 \), then \( \lambda^0 = 1, \lambda^k = 0, k > 0 \). The total TD loss, computed using cross-entropy, \( H \), is:

\[ L = - \sum_{t=1}^{T} H(y_t^\lambda ; y_t) \]

Figures A.3a and A.4a show the asymptotic accuracy versus TD(\( \lambda \)) for CIFAR-10 (5 trials per \( \lambda \)) and TinyImageNet (3 trials per \( \lambda \)), respectively. Table A.2 shows the full tabulated results for CIFAR-10, CIFAR-100, and TinyImageNet. Note, 5 trials per \( \lambda \) were trained for CIFAR-100.

A.3. Data Augmentation

When training **Sequential** and **Cascaded TD** models on CIFAR-10 and CIFAR-100, for each batch the \( 32 \times 32 \) images are padded with 4 pixels to each boarder (via reflection padding), resulting in a \( 40 \times 40 \) image. A random \( 32 \times 32 \) crop is taken, the image is randomly flipped horizontally, and standard normalized using the training set statistics is applied. Finally, a random \( 8 \times 8 \) block cut is taken such that the cropped pixels are set to 0. Images at run-time are only standard normalized using training set statistics - no other augmentation is applied with the exception of the persistent and transient noise experiments (see §4.4). The same process is followed for TinyImageNet with the following exceptions: (1) the \( 64 \times 64 \) images are padded to \( 86 \times 86 \) with reflection padding, random cropped back to \( 64 \times 64 \), randomly flipped horizontally, then standard normalized, and (2) no \( 8 \times 8 \) block cutting is applied.

A.4. Computing Infrastructure

We used 8x NVIDIA Telsa V100’s on Google Cloud Platform (GCP) for training all **Sequential** and **Cascaded TD** models; a single V100 was used for all evaluations, and to train **MetaCog** models. All models were implemented in PyTorch v1.5.0, using Python 3.7.7 operating on Ubuntu 18.04.

A.5. Average Runtime and Reproducibility

Table A.1 shows average runtimes (in hours) and ± SEM for training **Sequential** and **Cascaded TD** models on
each dataset. Reproducibility was ensured in the data pipeline and model training by seeding Random, NumPy, and PyTorch packages, as well as flagging deterministic cudnn via PyTorch API. When sweeping over \( \lambda \) for a given model and dataset, 5 trials, \([1, 2, 3, 4, 5]\), were trained to obtain statistical measures; the trials map to seeds \([42, 542, 1042, 1542, 2042]\). All models of same architecture used identical weights upon initialization. The average runtime for training MetaCog models on a single V100 GPU requires less than 3 minutes. When training multiple trials for a given MetaCog model, all models are initialized with the same weights, and 42 was used to seed all packages as detailed above.

A.6. Additional Temporal Dynamics Results

We show low and high selection latency instances between CascadeTD and CascadeSeq models in Figure A.2 for CIFAR-10. As with CIFAR-100, the qualitative differences between low and high selection latency for CascadeTD are stark, with low selection latency instances being more representative of prototypical instances of the given class (e.g., boats on blue water; horses in fields), whereas high selection latency instances are less typical (e.g., boats on green grass; horses in snow). Furthermore, we do not observe the strong delineation between low and high selection latency groups for CascadeSeq, supporting the claim that TD training allows the model to more rapidly respond to prototypical exemplars.

B. Meta-cognitive Experiment Details

For all meta-cognitive experiments, training data is generated from the EWS kernel applied to CascadeTD(0).

B.1. OOD Detection Dataset Details

CIFAR-10 serves as the in-distribution dataset, which contains 5,000 validation and 10,000 test set instances. The 5,000 validation instances, which we use as the in-distribution training set for OOD, were derived from a 90-10 train-validation split of the original 50,000 training instances used for training the CascadeTD model. The OOD datasets are as follows:

**TinyImageNet** The Tiny ImageNet (TinyImageNet) is a 200-class subset of ImageNet (Deng et al., 2009) and it contains 10,000 validation and 10,000 test instances. Following the methods of Liang et al. (2017) we introduce two variations: 1) resize; the image is downscaled to \(32 \times 32\), and 2) crop; a random \(32 \times 32\) crop is taken from the image.

**LSUN** The Large-scale Scene UNderstanding (LSUN) (Yu et al., 2015) consists of 10 scenes categories, such as classroom, restaurant, bedroom, etc. It contains 10,000 validation and 10,000 test instances, and similar to TinyImageNet, we use the resize and crop variations.

**SVHN** The Street View House Numbers (SVHN) (Netzer et al., 2011) dataset is obtained from house numbers in Google Street View images. It consists of 73,257 validation and 26,032 test set images.

B.2. OOD Detection

The MetaCog model is trained for 300 epochs with batch sizes of 256. We used Adam with an initial learning rate of 0.001 and weight decay of 0.0005 to optimize a binary
Training cascaded networks for speeded decisions using a temporal-difference loss

Table A.2. TD(λ) experiments. Green font indicates best performance across TD(λ) and SEQUENTIAL models for a given dataset. Highlight indicates best performing TD(λ) across λ’s for a given dataset.

<table>
<thead>
<tr>
<th>Dataset</th>
<th>TD(0)</th>
<th>TD(0.1)</th>
<th>TD(0.25)</th>
<th>TD(0.5)</th>
<th>TD(0.83)</th>
<th>TD(0.9)</th>
<th>TD(1)</th>
<th>SEQUENTIAL</th>
</tr>
</thead>
<tbody>
<tr>
<td>CIFAR-10</td>
<td>91.22 ± 0.18</td>
<td>91.48 ± 0.14</td>
<td>91.65 ± 0.08</td>
<td>91.45 ± 0.16</td>
<td>90.98 ± 0.21</td>
<td>90.07 ± 0.24</td>
<td>88.75 ± 0.42</td>
<td>91.91 ± 0.08</td>
</tr>
<tr>
<td>CIFAR-100</td>
<td>67.48 ± 0.14</td>
<td>67.41 ± 0.09</td>
<td>67.35 ± 0.20</td>
<td>67.00 ± 0.18</td>
<td>65.06 ± 0.11</td>
<td>63.90 ± 0.33</td>
<td>63.20 ± 0.14</td>
<td>65.56 ± 0.06</td>
</tr>
<tr>
<td>TinyImageNet</td>
<td>50.74 ± 0.11</td>
<td>51.60 ± 0.17</td>
<td>52.03 ± 0.07</td>
<td>52.25 ± 0.07</td>
<td>51.39 ± 0.13</td>
<td>50.84 ± 0.07</td>
<td>49.86 ± 0.15</td>
<td>52.46 ± 0.06</td>
</tr>
</tbody>
</table>

Table B.3. CIFAR-10 (in-distribution) vs. Aggregate OOD dataset quantitative measures corresponding to Figure 7. Each representation may include all time step outputs, \( t_{\text{all}} \), or only the final output, \( t_{\text{final}} \).

<table>
<thead>
<tr>
<th>OOD Representation</th>
<th>AUROC</th>
<th>FPR @ 95% TPR</th>
</tr>
</thead>
<tbody>
<tr>
<td>CASCADED + TD [MSP]</td>
<td>89.5 ± 0.5</td>
<td>63.0 ± 3.5</td>
</tr>
<tr>
<td>META + COG ( t_{\text{final}} ) [MSP]</td>
<td>88.8 ± 0.5</td>
<td>63.0 ± 3.1</td>
</tr>
<tr>
<td>META + COG ( t_{\text{all}} ) [MSP]</td>
<td>90.2 ± 0.5</td>
<td>46.4 ± 3.1</td>
</tr>
<tr>
<td>META + COG ( t_{\text{final}} ) [Entropy]</td>
<td>90.5 ± 0.3</td>
<td>51.2 ± 2.5</td>
</tr>
<tr>
<td>META + COG ( t_{\text{all}} ) [Entropy]</td>
<td>92.7 ± 0.3</td>
<td>38.9 ± 2.5</td>
</tr>
<tr>
<td>META + COG ( t_{\text{final}} ) [Softmax]</td>
<td>92.6 ± 0.1</td>
<td>31.7 ± 0.7</td>
</tr>
<tr>
<td>META + COG ( t_{\text{all}} ) [Softmax]</td>
<td>95.7 ± 0.1</td>
<td>20.5 ± 0.7</td>
</tr>
<tr>
<td>META + COG ( t_{\text{final}} ) [Logits]</td>
<td>96.7 ± 0.1</td>
<td>17.5 ± 0.4</td>
</tr>
<tr>
<td>META + COG ( t_{\text{all}} ) [Logits]</td>
<td>97.3 ± 0.1</td>
<td>13.7 ± 0.4</td>
</tr>
</tbody>
</table>

Cross entropy loss. Dropout with keep probability 0.5 was used for regularization. Numerics corresponding to Figure 7 are tabulated in Table B.3 with reported SEM corrected to remove random variance (Masson & Loftus, 2003).

B.3. Response Initiation

The META + COG GRU is trained for 300 epochs with a batch size of 256. We used Adam with an initial learning rate of 0.0001 and weight decay of 0.0001 to optimize a binary cross entropy loss.

The META + COG model is trained on the 4,500 instances of the CIFAR-10 validation set that have been processed by the CASCADED TD model, yielding a training set of dimension 4,500 × 70 × 10, where there are 70 timesteps and 10 logit values. We generate our evaluation set from the same method above using the CASCADED TD model on the 10,000 instance test set.

Figure A.5 shows the response initiation results comparing CASCADED + META + COG GRU with CASCADED SEQUENTIAL. Similar to the results for CASCADED TD + META + COG GRU versus CASCADED TD, the META + COG approaches yields significant improvements to response initiation.