Lab 5 - Ensembles et SVMs

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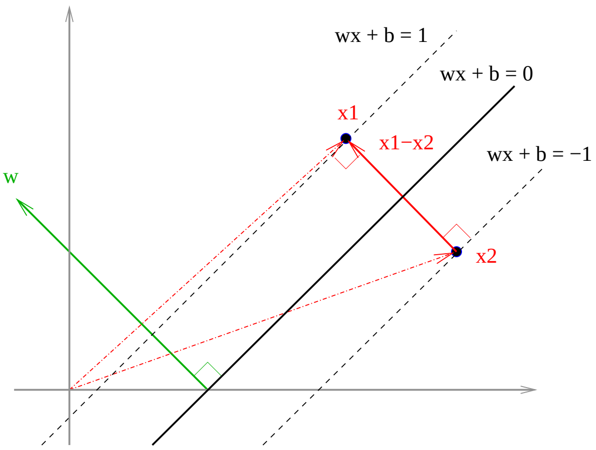
SVMs

1. **Show that the margin of a Support Vector Machine is $\frac{2}{\|w\|}$ wide.**

. . . .



Let $\mathbf{x_1}$ a point on the line with equation:

$$\mathbf{w} \cdot \mathbf{x} + b = 1, \tag{1}$$

and $\mathbf{x_2}$ the mirror point with respect to the classification hyperplane on the line with equation:

$$\mathbf{w} \cdot \mathbf{x} + b = -1 \tag{2}$$

As illustrated in the figure, the margin $\rho = \|\mathbf{x_1} - \mathbf{x_2}\|$.

$$(1) - (2)$$

$$\mathbf{w} \cdot \mathbf{x_1} + b - \mathbf{w} \cdot \mathbf{x_2} - b = 1 + 1$$

$$\mathbf{w} \cdot (\mathbf{x_1} - \mathbf{x_2}) = 2$$
$$\|\mathbf{w}\|\|\mathbf{x_1} - \mathbf{x_2}\| \cos(\mathbf{w}, \mathbf{x_1} - \mathbf{x_2}) = 2$$
$$\rho = \|\mathbf{x_1} - \mathbf{x_2}\| = \frac{2}{\|\mathbf{w}\|}$$

. . . .

2. **How would you control the capacity of an SVM?**

Ensembles

3. **Implementing bagging.**
   (a) Implement the functions defined in `bag.py` in order to make `bagtest.py` work.
   (b) Test with your favorite dataset (prefer a small one to debug).

. . . .

See `bag_solution.py`.

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4. **Show how bagging estimator tends to decrease the variance of a regressor learned using mean squared loss function.**
   (Hint: cf lab1)

. . . .

Let say that instead of bootstraps, we have $B_1, \ldots, B_b, \ldots$, being M training sets generated by the same distribution $\mathcal{P}$. Let $h(X; B)$ be the regressor optimized over B, and

$$h_A(X) = E_B[h(X; B)] = \frac{1}{M} \sum_B h(X; B)$$

the aggregated estimator of the regressor. Over the distribution of any new data point X, the average variance of the predictors $h(X; B)$ is:

$$s^2 = E_B E_X \left[(h(X; B) - E_X [h(X; B)])^2 \right] = E_X E_B \left[(h(X; B) - E_X [h(X; B)])^2 \right].$$

The variance of the aggregated predictor is:

$$s_A^2 = E_X \left[(h_A(X) - E_X [h_A(X)])^2 \right] = E_X \left[(E_B \{h(X; B) - E_X [h(X; B)]\})^2 \right]$$

We know that $(E[Z])^2 \leq E[Z^2]$, thus:

$$s_A^2 \leq s^2$$